

Large-Scale Schrödinger-Cat States and Majorana Bound States in Coupled Circuit-QED Systems

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We have studied the low-lying excitations of a chain of coupled circuit-QED systems in the ultrastrong coupling regime, and report several intriguing properties of its two nearly degenerate ground states. The ground states are Schrödinger cat states at a truly large scale, involving maximal entanglement between the resonators and the qubits, and are mathematically equivalent to Majorana bound states. With a suitable design of physical qubits, they are protected against local fluctuations and constitute a non-local qubit. Further, they can be probed and manipulated coherently by attaching an empty resonator to one end of the circuit-QED chain.

Confronted with formidable difficulties in solving strongly interacting many-body systems, it has been desired to find good quantum simulators. It may seem natural to simulate a many-body system with another tunable system of massive particles such as ultracold atomic gases [1]. In fact, any controllable quantum system, notably quantum computer if ever practical, can simulate efficiently many-body systems [2]. Indeed it has been recognized that photons confined in coupled-cavities simulate closely the quantum behaviors of strongly-correlated many-body systems [3, 4]. Subsequent studies have revealed that Bose-Hubbard model [5], interacting spin models [4, 6], and other exotic quantum phases [7] can be simulated efficiently using the coupled-cavities. Further, recent advances in solid-state devices such as circuit-QED systems [8, 9] and micro-cavities [10, 11] and ongoing efforts to fabricate large-scale cavity arrays [12] make the array of coupled cavities a promising candidate for an efficient quantum simulator.

Meanwhile, the ultrastrong coupling regime of the cavity-QED system, where the light-matter coupling energy is comparable to or even higher than the energy of the cavity field, has been envisioned [13] and experimentally demonstrated [14]. The ultrastrong coupling brings about fundamentally different physics deeply connected to the high degree of entanglement between the “matter” and the photon [15–19]. However, the effect of ultrastrong coupling on the low-energy excitations of an array of coupled cavity-QED systems remains unclear, and is our main concern in this work.

In this paper, we investigate the low-lying excitations of a one-dimensional (1D) array of circuit-QED systems (cQEDs), with each cQED being in the ultrastrong coupling regime; see Fig. 1. It turns out that the array permits two nearly degenerate ground states separated by a finite energy gap from the continuum of higher-energy states. We find several intriguing properties of the two ground states: (i) They are Schrödinger cat states at a truly large scale, and involve maximal entanglement between the resonators and the qubits. (ii) With a suitable design of physical qubits, the two ground states are protected against local fluctuations and constitute a non-local qubit [20]. (iii) They are mathematically equivalent to the long-sought Majorana bound states [21]. (iv) They can be probed and manipulated coherently by attaching an empty resonator to one end of the circuit-QED chain. Such configuration turns the

total system (the circuit-QED chain plus the empty resonator) into another effective circuit-QED system. There are many promising types of superconducting qubits, among which we focus on Fluxonium [22]. As we illustrate below, its strong inductive coupling with the superconducting resonator [17] and its anisotropic noise characteristics [18] are well suited for our purpose.

System: a circuit-QED chain — We consider a 1D array of cQEDs; see Fig. 1. Each cQED consists of the “resonator”, a superconducting microwave transmission line, and the “qubit”, a superconducting quantum bit (two-level system) [8, 23], and is theoretically described by the Rabi Hamiltonian

$$H_i^{\text{cQED}} = \omega_0 a_i^\dagger a_i - \lambda(a_i + a_i^\dagger)\sigma_i^x + \frac{\Omega}{2}\sigma_i^z \quad (1)$$

where a_i and a_i^\dagger are the field operators of the resonator with frequency ω_0 , σ_i^x and σ_i^z Pauli operators of the qubit with energy splitting Ω , and λ the resonator-qubit coupling energy in the i th cQED. The resonators of neighboring cQEDs are coupled capacitively to each other, and photons hop from one resonator to nearby ones. The Hamiltonian of the whole chain is thus given by

$$H = \sum_i^N H_i^{\text{cQED}} - J \sum_i^{N-1} (a_i^\dagger a_{i+1} + a_i a_{i+1}^\dagger) \quad (2)$$

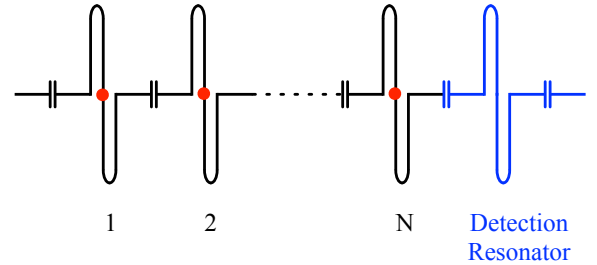


FIG. 1. Schematic of 1D circuit-QED arrays. The red dots indicate qubits placed inside of superconducting resonator. The N th resonator is coupled to the detection resonator. The circuit-QED array realizes the transverse field Ising model (TFIM), and the detection resonator can measure and control the degenerate ground state of the TFIM.

where J is the photon hopping amplitude and N is the number of cQEDs in the chain.

Before discussing the energy levels and associated wavefunctions of the whole chain, we first briefly review the properties of the low-lying states of a single cQED in the ultra-strong coupling regime ($\lambda \gtrsim \omega_0$). The strong coupling disables the standard rotating wave approximation, which reduces Eq. (1) to the Jaynes-Cummings Hamiltonian. As a consequence, the ground state of Eq. (1) is not a simple vacuum anymore as in the Jaynes-Cumming model. Instead, it contains finite average photon numbers, and shows non-classical properties such as squeezing and entanglement [15, 16]. To see this, let us examine the ground-state wavefunction more closely: Approximate expressions for the nearly-degenerate ground states have been derived in Ref. 17 (see also Ref. 16). Here we take a different approach and explore the parity symmetry in the Rabi Hamiltonian, which is important to understand the effect of photon hopping. The Hamiltonian (1) commutes with the “parity” operator $\Pi_i = \exp(-i\pi a_i^\dagger a_i) \sigma_i^z$, and thus the Hilbert space is classified into subspaces \mathcal{E}_i^\pm of \pm parity. Within each subspace \mathcal{E}_i^\pm , the Hamiltonian can be described in effect by a single bosonic operator, $b_i = a_i \sigma_i^x$: $H_i^{\text{cQED}} \rightarrow H_i^\pm = H_i^0 \pm H_i^1$ with $H_i^0 = \omega_0(b_i^\dagger - \lambda/\omega_0)(b_i - \lambda/\omega_0) - \lambda^2/\omega_0$ and $H_i^1 = \frac{\Omega}{2} \cos(\pi b_i^\dagger b_i)$ [16]. H_i^0 is simply a displaced harmonic oscillator and the ground state is a coherent state $|\lambda/\omega_0\rangle_{b_i}^\pm$. For $\lambda/\omega_0 \gg 1$ (regardless of Ω), H_i^1 can be treated perturbatively and shifts the energies of $|\lambda/\omega_0\rangle_{b_i}^\pm$ relatively by an exponentially small amount $\Delta = \frac{\Omega}{2} e^{-2(\lambda/\omega_0)^2}$. Now, back in the $\{a_i, \sigma_i^z\}$ -basis, the nearly degenerate ground states $|\lambda/\omega_0\rangle_{b_i}^\pm$ are expressed as

$$|0\rangle_i \equiv \frac{1}{\sqrt{2}} (|\lambda/\omega_0\rangle_i |+\rangle_i - |-\lambda/\omega_0\rangle_i |-\rangle_i), \quad (3a)$$

$$|1\rangle_i \equiv \frac{1}{\sqrt{2}} (|\lambda/\omega_0\rangle_i |+\rangle_i + |-\lambda/\omega_0\rangle_i |-\rangle_i), \quad (3b)$$

where $|\alpha\rangle_i$ ($\alpha \in \mathbb{C}$) is the eigenstate (coherent state) of a_i and $|\pm\rangle_i$ are the eigenstates of σ_i^x . In short, these two ground states, $|0\rangle_i$ and $|1\rangle_i$, residing in distinct parity subspaces are nearly degenerate with an energy splitting of 2Δ , separated far from higher-energy states by an energy gap ω_0 .

Effective model: a transverse-field Ising chain — Let us now investigate the whole chain described by the Hamiltonian (2). Typically $J \ll \omega_0$, and we are mainly interested in the low-lying excitations, well below ω_0 . In this limit, each cQED remains within the subspace spanned by the states $|0\rangle_i$ and $|1\rangle_i$ in Eq. (3) and can be regarded as a *pseudo-spin*:

$$\sum_i^N H_i^{\text{cQED}} = -\Delta \sum_i^N \tau_i^z \quad (4)$$

where $\tau_i^z = |0\rangle_i \langle 0| - |1\rangle_i \langle 1|$ and the energy splitting Δ plays the role of Zeeman field. Hopping of a photon into or out of a cavity changes the parity of its state, or more explicitly $a_i |0\rangle_i = \lambda/\omega_0 |1\rangle_i$ and $a_i |1\rangle_i = \lambda/\omega_0 |0\rangle_i$. Based on these

observation, we can identify a_i and a_i^\dagger as a pseudo-spin-flip operator τ_i^x and $a_i a_{i+1}^\dagger$ as Ising interaction $\tau_i^x \tau_{i+1}^x$. That is, the photon-hopping part of the Hamiltonian becomes

$$J \sum_i^{N-1} (a_i^\dagger a_{i+1} + a_i a_{i+1}^\dagger) = J_{\text{eff}} \sum_i^{N-1} \tau_i^x \tau_{i+1}^x \quad (5)$$

with $J_{\text{eff}} = 2J(\lambda/\omega_0)^2$. The effective Ising interaction strength, J_{eff} , is renormalized with respect to J by the factor $(\lambda/\omega_0)^2$ because the field part of the pseudo-spin states in Eq. (3) is a coherent state with amplitudes λ/ω_0 and the field-field interaction between resonators is proportional to the amplitudes of the resonator fields.

Putting both terms in Eqs. (4) and (5) together, the low-energy effective Hamiltonian for the cQED chain becomes the so-called transverse-field Ising model (TFIM),

$$H_{\text{Ising}} = -\Delta \sum_i^N \tau_i^z - J_{\text{eff}} \sum_i^{N-1} \tau_i^x \tau_{i+1}^x. \quad (6)$$

The TFIM exhibits a quantum phase transition between the *magnetically ordered* phase for $\Delta < J_{\text{eff}}$ and the *quantum paramagnet* phase for $\Delta > J_{\text{eff}}$ [24]. The former is particularly interesting for our purposes. For $\Delta = 0$, H_{Ising} has two degenerate ground states, $|\Rightarrow\rangle \equiv \prod_i |\rightarrow\rangle_i$ and $|\Leftarrow\rangle \equiv \prod_i |\leftarrow\rangle_i$, where $|\rightarrow\rangle_i$ and $|\leftarrow\rangle_i$ are eigenstates of τ_i^x . For $\Delta > 0$ (yet $\Delta < J_{\text{eff}}$), τ_i^z tends to flip the pseudo-spins, $|\rightarrow\rangle_i \leftrightarrow |\leftarrow\rangle_i$. It causes tunneling between $|\Rightarrow\rangle$ and $|\Leftarrow\rangle$ via soliton propagation, and hence the true eigenstates become

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} (|\Rightarrow\rangle + |\Leftarrow\rangle), \quad |\Psi_1\rangle = \frac{1}{\sqrt{2}} (|\Rightarrow\rangle - |\Leftarrow\rangle) \quad (7)$$

However, as the tunneling involves N spins, the tunneling amplitude is exponentially suppressed with the system size N . In other words, $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are nearly degenerate with energy splitting, $\delta \sim \exp(-N/\xi)$ with ξ being the correlation length of the Ising chain, exponentially small in system size N . Both are separated from the continuum of excitations by the energy gap J_{eff} .

The two states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ in Eq. (7) have non-local combinations of many pseudo-spins and are widely known as Greenberger-Horne-Zeilinger (GHZ) states [25]. Moreover, by expressing them in the original $\{a_i, \sigma_i^x\}$ -basis

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}} \left[\prod_i^N |\lambda/\omega_0\rangle_i |+\rangle_i + (-1)^s \prod_i^N |-\lambda/\omega_0\rangle_i |-\rangle_i \right] \quad (8)$$

with $s = 0$ or 1 , one can see that they involve high degree of non-local entanglement between cavity fields and qubits. They are thus *Schrödinger cat states* at a truly large scale while many theoretically proposed or experimentally demonstrated Schrödinger cat states [26, 27] contain merely a single radiation field. Below we illustrate that the two states in (8) are protected against local fluctuations and constitute a non-local qubit [20].

Effective model: a Majorana chain — 1D TFIM discussed above is equivalent to a chain of Majorana fermions [28, 29]. The latter has attracted great interest because it permits localized Majorana modes that can be used for topologically protected quantum computation [28–30]. A very recent experiment [31] suggests that the Majorana chain can be realized in a solid-state system, and intensive efforts are made in this direction [32].

Here we re-express the two nearly degenerate states in Eq. (7) or (8) in terms of localized Majorana fermions, and later discuss an experimentally feasible way of probing such Majorana fermions. The equivalence between the TFIM and the Majorana chain can be seen through a Jordan-Wigner transformation [33]: $c_i^\dagger = \tau_i^+ \prod_{j=1}^{i-1} (-\tau_j^z)$ with $\tau_i^+ = \frac{1}{2}(\tau_i^x + i\tau_i^y)$. The operators c_i and c_i^\dagger describe Dirac fermions and satisfy $\{c_i, c_j^\dagger\} = \delta_{ij}$ and $\{c_i, c_j\} = 0$. The Dirac fermion operators are further represented with self-conjugate Majorana operators, $\gamma_{2i-1} = c_i^\dagger + c_i$ and $\gamma_{2i} = i(c_i^\dagger - c_i)$. The TFIM (6) is then reduced to

$$H_{\text{Majorana}} = \frac{i}{2} \left[\Delta \sum_{i=1}^N \gamma_{2i-1} \gamma_{2i} + J_{\text{eff}} \sum_{i=1}^{N-1} \gamma_{2i} \gamma_{2i+1} \right] \quad (9)$$

At $\Delta = 0$, the Majoranas at the two ends, γ_1 and γ_{2N} , in the chain does not appear in the Hamiltonian, which implies the existence of two degenerate ground states. These are nothing but $|\Rightarrow\rangle$ and $|\Leftarrow\rangle$ in Eq. (7). For finite Δ , the two states $|\Rightarrow\rangle$ and $|\Leftarrow\rangle$ are mixed linearly into $|\Psi_0\rangle$ and $|\Psi_1\rangle$ in Eq. (7) due to the tunneling between two Majorana modes γ_1 and γ_{2N} , and the degeneracy is lifted. Since the tunneling is through the whole chain, the energy splitting δ is exponentially small (as long as $\Delta < J_{\text{eff}}$). One can check that $(\gamma_1 + i\gamma_{2N})|\Psi_0\rangle = 0$ and $(\gamma_1 + i\gamma_{2N})|\Psi_1\rangle = 2|\Psi_0\rangle$, which means that $|\Psi_1\rangle$ has one more fermion than $|\Psi_0\rangle$ or equivalently that $|\Psi_0\rangle$ and $|\Psi_1\rangle$ have different fermion parities.

Here we emphasize that the two Majoranas localized at the ends of the Majorana chain are actually non-local in the physical chain, i.e., the cQED chain or the Ising chain [21]: The Majorana operators is represented in terms of τ_j^x and τ_j^z as

$$\gamma_1 = \tau_1^x, \quad \gamma_{2N} = i\tau_N^x \prod_{j=1}^N (-\tau_j^z), \quad (10)$$

and γ_{2N} involves the *string* operator $\prod_{j=1}^N (-\tau_j^z)$. This implies that the two nearly-degenerate ground states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are not protected topologically against local noise even though mathematically they correspond to two distinct Majorana modes. It is in stark contrast to the case where the two Majorana modes at the ends of a p -wave superconducting wire are topologically protected. However, we will see below that the two states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are vulnerable only to a certain type of local noise and there exist realistic systems with such type of local noise significantly suppressed.

Noise — It is evident from the expression in Eq. (8) that the non-local spin qubits are prone to the local noise in σ_i^x of

the physical qubits and the one in $a_i + a_i^\dagger$ of the resonators. The states are intrinsically robust against the σ_i^y and σ_i^z noise since ${}_i\langle\Psi_1|\sigma_i^{y,z}|\Psi_0\rangle_i \sim e^{-(\lambda/\omega_0)^2}$, which is reminiscent of the Franck-Condon effect. The $a_i + a_i^\dagger$ noise affects only the resonator at the end of the chain (which is usually connected external microwave environment for measurement), and can be easily avoided by replacing it by a high-Q resonator. The problem with σ_i^x noise can be circumvented, for example, by using Fluxonium for qubits. Fluxonium is known to have anisotropic noise characteristics with $\sigma_i^{y,z}$ being the dominant noises and the σ_i^x noise ignorable [18].

What about the effect of inhomogeneity in system parameters? Above we have assumed ω_0 , Ω , λ and J of each circuit-QED to be homogeneous. Deviations in ω_0 , Ω , λ lead to fluctuations in Δ . The inter-cavity coupling strength, J , can also be varied from cavity to cavity, which leads to inhomogeneous TFIM,

$$- \sum_i \Delta_i \tau_i^z - \sum_i J_i^{\text{eff}} \tau_i^x \tau_{i+1}^x. \quad (11)$$

This Hamiltonian still conserves the parity symmetry, $\mathcal{P} = \prod_{i=1}^N \tau_i^z$ which is respected by the degenerate ground states. Therefore, the ground states will be robust to small fluctuations in Δ_i and J_i . We thus conclude that the nearly degenerate ground states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ can be kept well protected by a careful design of the physical qubits in the system.

Detection and control — In this section, we suggest a scheme to control and measure the non-local spin qubit. It can be also interpreted as detecting the Majorana bound states. Our proposal consists only of an additional empty resonator coupled to the resonator at the end of the circuit-QED chain. Consider a resonator with a frequency, ω_d , capacitively coupled to N th cavity, so that we have

$$H_d = J_d(a_N^\dagger a_d + a_N a_d^\dagger) + \omega_d a_d^\dagger a_d \quad (12)$$

where a_N represents the field operator of N th cavity, and a_d the field operator of the detection cavity. As shown earlier, the N th cavity's creation and annihilation operators are equivalent to $\lambda/\omega_0 \tau_N^x$ for the N th effective spin. Moreover, for the non-local spin qubits, τ_i^x is equivalent to $S^x = |\Psi_0\rangle\langle\Psi_1| + |\Psi_1\rangle\langle\Psi_0|$ for any i as $\tau_i^x |\Psi_s\rangle = |\Psi_{1-s}\rangle$ ($s = 0, 1$). Therefore, assuming that $\tilde{J}_d \equiv J_d \lambda/\omega_0 \ll J_{\text{eff}}$, the low-energy effective Hamiltonian (6) combined with the detection Hamiltonian (12) leads again to the Rabi Hamiltonian

$$H_{\text{Rabi}} = \frac{\delta}{2} S^z + \tilde{J}_d S^x (a_d + a_d^\dagger) + \omega_d a_d^\dagger a_d \quad (13)$$

Here we can make the rotating wave approximation, then the Hamiltonian reduces to the Jaynes-Cummings Hamiltonian. Therefore, by just adding an empty resonator at one end of the circuit-QED array, we can realize a circuit-QED Hamiltonian for the non-local spin qubit. It allows us to tap into the standard techniques available for the circuit-QED to control and measure the non-local spin qubit. For example, since the

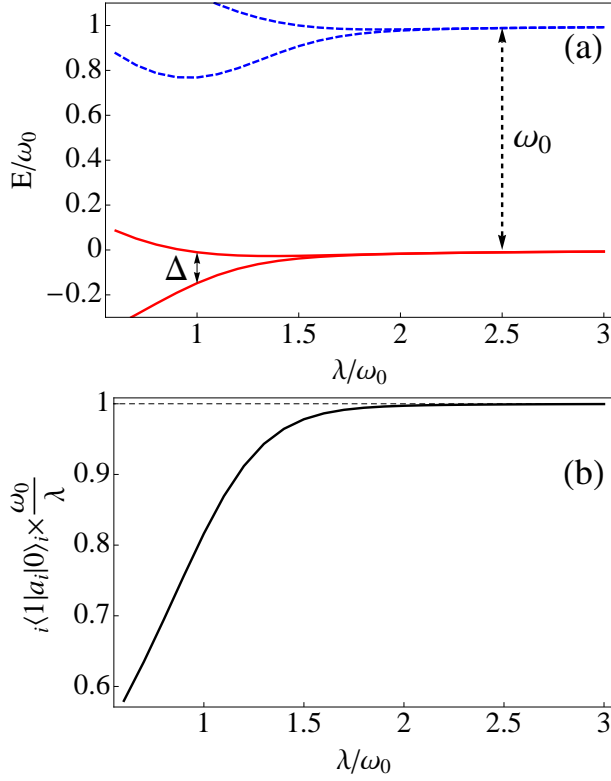


FIG. 2. (a) Energy diagram for the circuit-QED Hamiltonian (1) as a function of λ/ω_0 . (b) Plot of $\frac{\omega_0}{\lambda} \langle 1|a_i|0 \rangle_i$. We can conclude that $\lambda > 2\omega_0$ is required for our model to be valid because the transverse field Δ almost vanishes and the identification of photon annihilation operator as a spin flip operator, $\frac{\omega_0}{\lambda} a_i = \tau_i^x$, is justified.

detuning between the detection cavity frequency and the non-local spin qubit splitting, $\Delta_d = \omega_d - \delta$ is large compared to \tilde{J}_d , it is in the dispersive regime where the cavity frequency pulling by the non-local spin qubit is $\delta\omega_d = 2J_d^2/\Delta_d$ [23]. This can be experimentally measured since one can have $\tilde{J}_d \sim 10^{-4}\omega_0$ which achieves the standard strong-coupling regime for the circuit QED [8].

Experimental feasibility — Finally we examine the experimental feasibility of the ideas explained above, estimating possible values of physical parameters of the system. Two requirements must be satisfied: First, the two ground states of each cQED in the system must be nearly degenerate and well separated from higher excitations. In Fig. (2) (a) are plotted the energies of individual circuit-QED Hamiltonian (1) in the resonant case ($\omega_0 = \Omega$). Figure 2 (b) plots $\frac{\omega_0}{\lambda} \langle 1|a_i|0 \rangle_i$ to illustrate how good (its value close to 1) the approximation $a_i = \lambda/\omega_0 \tau_i^x$ is. One can see that $\lambda \sim 2\omega_0$ suffices for the requirement. Second, the system should be in the magnetically ordered phase (in terms of the effective TFIM), $\Delta < J_{\text{eff}}$ or equivalently $\Omega \exp[-2(\lambda/\omega_0)^2] < 4J(\lambda/\omega_0)^2$. This requirement is satisfied provided that $J > 10^{-5}\omega_0$. The desired coupling strength, $\lambda > 2\omega_0$, seems achievable for the Fluxonium coupled inductively to the superconducting resonator [18]. Moreover, $J > 10^{-5}\omega_0$ is also realistic for the

superconducting resonators, with J in the range of a few MHz.

Conclusion — We have found several intriguing properties of the two nearly degenerate ground states of a chain of coupled circuit-QED systems in the ultrastrong coupling regime. The ground states are Schrödinger cat states at a truly large scale, and are mathematically equivalent to Majorana bound states. With a suitable design of the system, they are protected against local fluctuations, and may be probed and manipulated coherently by attaching an extra empty resonator.

Finishing this work, we have noticed a closely related preprint [34]. While they focus on the phase transition of the circuit-QED chain, we are mainly concerned about the quantum properties of the nearly degenerated ground states on one side of the phase transition. In this respect, both works are complementary to each other.

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